

## **Mathematics Tutorial Series**

Integral Calculus #6

## **Basic Anti-derivatives**

Remember the Table of Key Derivatives:

| f                   | f'                       |
|---------------------|--------------------------|
| c, constant         | 0                        |
| $\chi^m$            | $mx^{m-1}$               |
| sin x               | cos x                    |
| cos x               | $-\sin x$                |
| tan x               | sec <sup>2</sup> x       |
| sec x               | sec x tan x              |
| e <sup>x</sup>      | e <sup>x</sup>           |
| $\log x$            | $\frac{1}{x}$            |
| $\sin^{-1} x$       | $\frac{1}{\sqrt{1-x^2}}$ |
| tan <sup>-1</sup> x | $\frac{1}{1+x^2}$        |

Switch the columns.

This becomes a Table of Key Anti-derivatives.

| f                        | Anti-derivative of $f$ |
|--------------------------|------------------------|
| 0                        | c, constant            |
| $mx^{m-1}$               | $\chi^m$               |
| $x^m$                    | $\frac{1}{m+1}x^{m+1}$ |
| cos x                    | sin x                  |
| $-\sin x$                | cos x                  |
| sec <sup>2</sup> x       | tan x                  |
| sec x tan x              | sec x                  |
| $e^x$                    | e <sup>x</sup>         |
| $\frac{1}{x}$            | $\log x$               |
| $\frac{1}{\sqrt{1-x^2}}$ | $\sin^{-1} x$          |
| $\frac{1}{1+x^2}$        | tan <sup>-1</sup> x    |

## **Examples:**

1.

$$\int \sec^2 x \, dx = \tan x + c$$

and so

$$\int_0^{\pi/4} \sec^2 x \, dx = \tan \frac{\pi}{4} - \tan 0 = 1 - 0 = 1$$

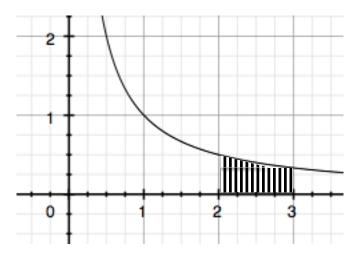
2.

$$\int \frac{1}{x} dx = \log x + c$$

and so

$$\int_{2}^{3} \frac{1}{x} \, dx = \log 3 - \log 2 = 0.4055$$

This number is the area under the graph of  $y = \frac{1}{x}$  between x = 2 and x = 3.



3.

$$\int 1 \, dx = x + c$$

## **Summary**

- 1. A definite integral can be calculated once we have an anti-derivative of the integrand.
- 2. What we know about derivatives can help us construct anti-derivatives.
- 3.  $\int f(x) dx$  is a function plus a constant.
- 4.  $\int_a^b f(x) dx$  is a number.